

# Canard Cycles And Center Manifolds

Canard explosion in delayed equations with multiple timescales

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**Abstract** We analyze canard explosions in delayed differential equations with a one-dimensional slow manifold. This study is applied to explore the dynamics of the van der Pol slow-fast system with delayed self-coupling. In the absence of delays, this system provides a canonical example of a canard explosion. We show that as the delay is increased a family of "classical" canard explosions ends as a Bogdanov-Takens bifurcation occurs at the folds points of the S-shaped critical manifold.

**Keywords** Delayed Differential Equations; Slow-Fast systems; Canard Explosion

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## Introduction

Nonlinear dynamical systems with multiple timescales and delays are essential in applications. For instance, realistic models of neuronal dynamics accounting for the dynamics of neuronal areas involve several excitable elements, whose dynamics occur on very different timescales, interacting after delays due to the transmission of information through synapses. Similar problems arise in different domains, including mechanical systems [2], macroscopic phenomena arising in chemistry, physics or social science. Such nonlinear systems involving multiple timescale dynamics and delays generally display a rich phenomenology, and particularly a wide repertoire of complex periodic behaviors. Slow-fast systems have been chiefly analyzed in finite-dimensional contexts. The topic of the present paper is to analyze the role of delays in dynamics of slow-fast systems.

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Canard Cycles and Center Manifolds cover image. Memoirs of the American Mathematical Society ; pp; Softcover MSC: Primary Statement of the result: The canard phenomenon for the singular Van der Pol equation; 2. Global desingularization; 3. Foliations by center manifolds; 4. Request PDF on ResearchGate Canard cycles and center manifolds In this paper the "canard phenomenon" occurring in Van der Pol's equation  $\ddot{x} + (x^2 + x)\dot{x} = \epsilon \dot{y}$ . Request PDF on ResearchGate Canard cycles and center manifolds / Freddy Dumortier, Robert Roussarie Volume , number (first of 4 numbers). Title, Canard Cycles and Center Manifolds, Issue Volume of American Mathematical Society: Memoirs of the American Mathematical Society Canard. The authors give a geometric explanation and proof of this phenomenon using foliations by center manifolds and blow-up of unfoldings as essential techniques. Canard Cycles and Center Manifolds. Front Cover. Freddy Dumortier, Robert H. Roussarie. American Mathematical Society, - Bifurcation theory - Canard Cycles And Center Manifolds by Freddy Dumortier, , available at Book Depository with free delivery worldwide. Get this from a library! Canard cycles and center manifolds. [Freddy Dumortier; Robert Roussarie]. Furthermore, the shape of a canard cycle in the phase space . to a one slow and one fast variable system by a center manifold reduction. Title: Canard cycles and center manifolds. Authors: DUMORTIER, Freddy Roussarie, R. Issue Date: Citation: Memoirs of the American Mathematical . Proofs are based on the techniques introduced in Canard Cycles and Center Manifolds (F. Dumortier and R. Roussarie, , Mem. Amer. proof of canard cycles in van der Pol's equation by using blow-up of singularities and foliation by center manifolds as main techniques. The method is very. canard cycles are created as the result of a Hopf bifurcation. as the slow nullcline passes through a corner of the critical manifold. [9] Freddy Dumortier and Robert H Roussarie, Canard cycles and center manifolds, vol. Mixed mode oscillations due to the generalized canard phenomenon. .. F. Dumortier and R. Roussarie, Canard cycles and center manifolds, . Memoirs of the. perbolic equilibria can be studied through center manifolds. . [3] Dumortier, F. and Roussarie, R. () Canard cycles and center manifolds.

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